

General Certificate of Education
June 2006
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Monday 22 May 2006 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

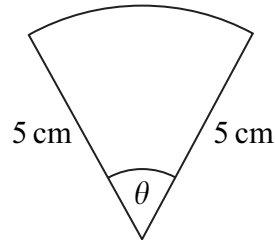
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

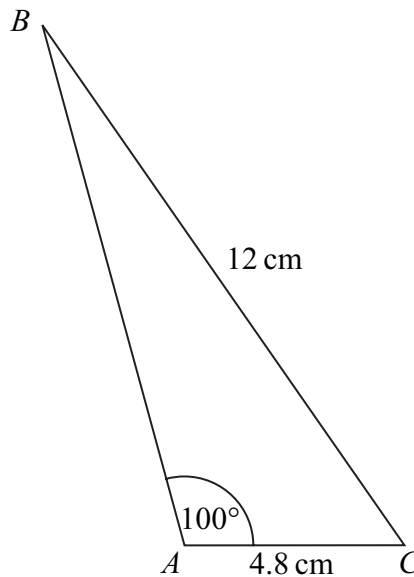
- 1 The diagram shows a sector of a circle of radius 5 cm and angle θ radians.



The area of the sector is 8.1 cm^2 .

- (a) Show that $\theta = 0.648$. (2 marks)
- (b) Find the perimeter of the sector. (3 marks)

- 2 The diagram shows a triangle ABC .



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

The size of the angle BAC is 100° .

- (a) Show that angle $ABC = 23.2^\circ$, correct to the nearest 0.1° . (3 marks)
- (b) Calculate the area of triangle ABC , giving your answer in cm^2 to three significant figures. (3 marks)

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n . *(2 marks)*

4 (a) The expression $(1 - 2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q .
(3 marks)

(b) Find the coefficient of x in the expansion of $(2 + x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$. *(3 marks)*

5 (a) Given that

$$\log_a x = 2 \log_a 6 - \log_a 3$$

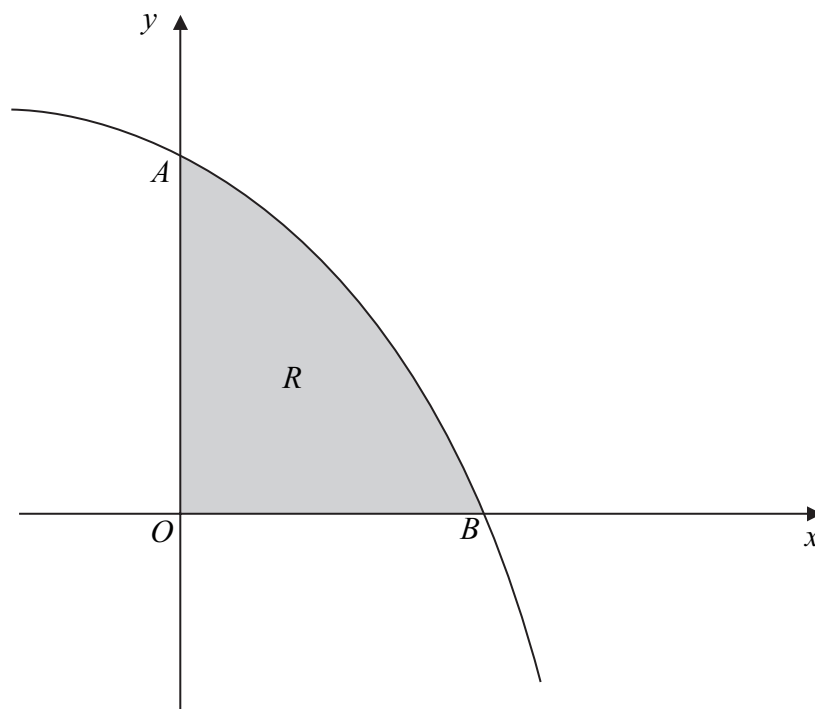
show that $x = 12$. *(3 marks)*

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express y in terms of a , giving your answer in a form not involving logarithms.
(3 marks)

- 6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y -axis at the point A and the x -axis at the point B .

- (a) (i) Find the y -coordinate of point A . (2 marks)
- (ii) Verify that the x -coordinate of point B is 3. (1 mark)
- (b) The region, R , bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R . (4 marks)
- (c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. (3 marks)
- (ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$. Find the value of k . (1 mark)
- (d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. (2 marks)
- (ii) Sketch the curve $y = -3^x$. (2 marks)

7 At the point (x, y) , where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 4$. (1 mark)

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. (1 mark)

(iii) Find $\frac{d^2y}{dx^2}$. (3 marks)

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. (2 marks)

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. (1 mark)

(ii) Find an equation of the normal to the curve at P . (3 marks)

(c) (i) Find $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7) dx$. (3 marks)

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. (3 marks)

8 (a) Describe the single geometrical transformation by which the curve with equation $y = \tan \frac{1}{2}x$ can be obtained from the curve $y = \tan x$. (2 marks)

(b) Solve the equation $\tan \frac{1}{2}x = 3$ in the interval $0 < x < 4\pi$, giving your answers in radians to three significant figures. (4 marks)

(c) Solve the equation

$$\cos \theta (\sin \theta - 3 \cos \theta) = 0$$

in the interval $0 < \theta < 2\pi$, giving your answers in radians to three significant figures. (5 marks)

END OF QUESTIONS

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